

From de Broglie Waves to Option Pricing: A Wave Mechanics Approach to Financial Derivatives

— Quantum Finance —

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A quantum inspired financial modelling

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Abstract

The Black–Scholes model, cornerstone of modern option pricing, is mathematically equivalent to a heat diffusion equation. It is well known that the heat equation is related to the Schrödinger equation of quantum mechanics through a Wick rotation — the substitution of real time by imaginary time. The existing literature on quantum finance has exploited this connection, but systematically eliminates the imaginary unit i in the process, thereby discarding the oscillatory and wave-like features that make quantum mechanics so distinctive.

In this paper, we take a different path. Inspired by Louis de Broglie’s original hypothesis that every particle carries an associated wave, we construct a *genuine* wave mechanics for financial assets, in which the imaginary unit i plays a structural role rather than being rotated away. We associate to each underlying asset a financial wave function $\psi(x, t)$ governed by a Schrödinger-type equation, and we show that wave phenomena (interference, tunnelling, diffraction and wave-function collapse) provide natural explanations for empirically observed market features such as the volatility smile, barrier option mispricing, false breakouts and post-announcement dynamics. We derive a closed-form pricing formula for European options that decomposes into classical Black–Scholes components plus an interference correction term, and we compute the full set of associated hedging sensitivities, including three novel “Greeks” that have no classical counterpart: the Planck Greek Ξ , the phase Greek Φ and the coherence Greek κ . We verify that the model reduces exactly to Black–Scholes in the appropriate classical limit.

Keywords: Option pricing, quantum finance, de Broglie waves, Schrödinger equation, volatility smile, wave interference, hedging Greeks, financial wave mechanics.

JEL Classification: G13, C65, G12.

8 Conclusion

This paper has proposed a wave-mechanical framework for option pricing that preserves the imaginary unit i as a structural element rather than rotating it away. Starting from a financial analogue of de Broglie's hypothesis, we have derived a financial Schrödinger equation, identified four wave phenomena with clear market interpretations, obtained a closed-form pricing formula for European options and computed the full set of hedging Greeks, including three novel sensitivities that capture wave-specific risks.

The model offers several conceptual advantages. The volatility smile emerges naturally as an interference pattern, without requiring ad hoc modifications to the underlying process. Barrier option mispricing is explained by quantum tunnelling. False breakouts are a manifestation of diffraction. Post-announcement dynamics are described by wave-function collapse. Each of these explanations carries a strong physical intuition that can aid practitioners in developing qualitative understanding of market phenomena.

The model introduces a small number of new parameters beyond Black–Scholes: the financial Planck constant $\hbar_f$, the wave number difference Δk , the weights (w_1, w_2) and the initial phase φ_0 . In principle, these can be calibrated from the observed volatility surface. The coherence factor $e^{-(\Delta k)^2 \sigma_{\text{eff}}^2 / 2}$ determines the amplitude of the smile oscillations, while the fringe spacing $2\pi/|\Delta k|$ determines their frequency in strike space. The phase φ_0 shifts the smile pattern horizontally. A systematic empirical study of this calibration, comparing the model's performance against standard alternatives (Dupire, Heston, SABR) is the most important direction for future work.

An empirically attractive feature is that $\hbar_f$ can be estimated as the gap between observed option prices and Black–Scholes prices. Markets known to be “more quantum” (less liquid, more informationally fragmented and more prone to microstructure effects) should exhibit larger values of $\hbar_f$.

The present treatment is deliberately simplified in several respects. We have considered only two interfering signals, whereas real markets may contain many. The extension to N signals is straightforward in principle (the interference becomes a sum over all pairs) but increases the number of parameters. We have treated the potential $V(x, t)$ only qualitatively. A systematic classification of financial potentials (barrier, harmonic, step, periodic) and their effects on option pricing is a natural next step. We have not addressed American options, path-dependent options or multi-asset options, each of which presents specific technical challenges (early exercise, history dependence, entanglement respectively) that deserve separate treatment.

Perhaps the most important conceptual limitation is the interpretation of $|\psi|^2$ as a risk-neutral density. In quantum mechanics, the Born rule ($|\psi|^2 = \text{probability density}$) is a postulate with deep experimental support. In finance, identifying $|\psi|^2$ with the risk-neutral density is an assumption that must be justified empirically. The classical limit analysis of Section 7 shows that this identification is at least consistent, but a deeper foundational discussion is warranted.

Despite these limitations, we believe that the wave-mechanical perspective opens a genuinely new direction in quantitative finance. The language of waves (interference, coherence, phase and tunnelling) provides an intuitive and mathematically rich framework for understanding market phenomena that lie beyond the reach of pure diffusion. Our hope is that this paper will encourage further exploration of this direction, both theoretical and empirical.