

Quantum DSGE: A de Broglie Framework for Macroeconomic Dynamics

— Quantum Economics —

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A quantum inspired macroeconomic modelling

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June 12, 2026

Abstract

This paper derives — rather than postulates — a quantum-mechanical representation of macroeconomic dynamics. The primitive is a standard heterogeneous-agent overlapping-generations economy à la Blanchard–Yaari: households face idiosyncratic risk σ_s , quadratic adjustment costs scaled by the capital stock K and generational turnover ν_g . Its equilibrium is a mean field game — a backward Hamilton–Jacobi–Bellman equation coupled to a forward Kolmogorov equation. An exact Cole–Hopf transformation maps this pair into a Schrödinger system with $\hbar_{eco} = \sigma_s^2 K$: Planck’s constant is the product of idiosyncratic variance and aggregate inertia, the wave function is $\Psi = \sqrt{m} e^{-iW/\hbar_{eco}}$ (cross-sectional distribution $\times$ value function), and the Born rule is an aggregation identity. The Smets–Wouters IS–Phillips–Taylor block is recovered as the *representative-agent limit* $\hbar_{eco} \rightarrow 0$, and the quantum corrections are the distributional (HANK) channel in closed form. Oscillations are an equilibrium property: generational turnover plus aggregate-demand feedback place the economy on the complex-eigenvalue branch of the linearized system.

As an application, the zero lower bound enters as an infinite potential wall, yielding closed-form central-bank indicators — a ZLB probability, a trap duration $\tau_{ZLB} = \pi/\omega_{eco}$ and a Quantum Secular Stagnation Index — while the OLG block endogenizes the drift towards the wall through aging and r^* . Calibrated on US FRED data (1990–2024), the model generates Kitchin, Juglar and Kuznets cycles as harmonics of a single $\omega_{eco} = 0.40$ rad/quarter and beats AR(1) forecasts out of sample by 29% RMSE at the 8-quarter horizon (Diebold–Mariano $p < 0.01$); an Unscented Kalman Filter tracks $(\hbar_{eco}, \omega_{eco}, s^*)$ in real time. Matching UN World Population Prospects 2024 demographics to World Bank GDP for 36 countries confirms the model’s two sharpest demographic predictions: the fundamental frequency is *universal* ($\bar{\omega} = 0.355$ rad/quarter, coefficient of variation 12%, no demographic gradient — as predicted, since demography enters ω_{eco} at second order), while aging loads the *slow* channel: on WPP2024 medium-variant projections, the demographic component of the ZLB probability reaches 47% in Korea and 28% in China by 2075, against 3% in the United States. A two-oscillator extension couples the business-cycle and demographic modes — the Easterlin mechanism as mode coupling — and detects its first spectral trace: a combination line at $\omega_J + \omega_d$ in the US output-gap spectrum ($p = 0.026$, placebo-clean).

Keywords: DSGE, wave function, quantum economics, overlapping generations, mean field games, heterogeneous agents, zero lower bound, secular stagnation, demography, Kalman filter, business cycles, Schrödinger equation.

12 Conclusion

This paper has rebuilt the Quantum DSGE from microfoundations. Our main findings are:

1. *The formalism is derived, not assumed.* A heterogeneous Blanchard–Yaari OLG economy — households with idiosyncratic risk σ_s , adjustment costs scaled by capital K , generational turnover ν_g — has a mean field game equilibrium whose exact Cole–Hopf image is a Schrödinger system with $\hbar_{\text{eco}} = \sigma_s^2 K$. The wave function is $\Psi = \sqrt{m} e^{-iW/\hbar_{\text{eco}}}$: amplitude = the cross-sectional distribution, phase = the value function. The Smets–Wouters block is the representative-agent limit $\hbar_{\text{eco}} \rightarrow 0$; the quantum corrections ($Q_{\text{IS}}, Q_{\text{PC}}, Q_{\text{Taylor}}$) are the distributional (HANK) channel in closed form; and oscillations arise endogenously when generational turnover and demand feedback push the linearized equilibrium onto its complex-eigenvalue branch — the continuous-time, heterogeneous-agent descendant of Grandmont’s OLG cycles.
2. *The ZLB block is an application, and a fruitful one.* The infinite-wall treatment yields three closed-form indicators ($\mathbb{P}(\text{ZLB})$, $\tau_{\text{ZLB}} = \pi/\omega_{\text{eco}}$, QSSI), and the OLG structure endogenizes the drift towards the wall: aging lowers r^* and loads the indicators through eq. (46). Calibrated on US FRED data (1990–2024), the model generates the three classical cycle frequencies as harmonics of $\omega_{\text{eco}} = 0.40$ rad/quarter, outperforms AR(1) forecasts by 29% RMSE at the 8-quarter horizon (DM $p < 0.01$), and supports a real-time UKF that detects regime shifts within two quarters.
3. *Demography enters exactly where the theory says it should.* On WPP2024 $\times$ World Bank data for 36 countries, the fundamental frequency is universal ($\bar{\omega} = 0.355$ rad/quarter, CV 12%) and shows no demographic gradient — the null predicted by the model, in which the demographic component of ω_{eco} is second-order. Birth series carry a genuine wave in the Kuznets band (61% of countries, median period 19 years), on a clock distinct from the business cycle. And the first-order demographic channel is the slow one: on medium-variant projections, the demographic component of $\mathbb{P}(\text{ZLB})$ reaches 47% in Korea and 28% in China by 2075, against $\sim 3\%$ in the United States and in a Japan that has already absorbed its demographic shock. Expected trap durations are similar everywhere ($\tau_{\text{ZLB}} \approx 8\text{--}9$ quarters); what aging changes is the frequency of collisions with the wall.
4. *The two clocks are coupled.* The two-oscillator extension of Section 11 recasts the Easterlin mechanism as mode coupling in a two-dimensional state space: cohort crowding tilts the macroeconomic potential ($\lambda_c s d$) and modulates the turnover, while procyclical relative income forces the birth oscillator. The coupling leaves the universality of ω_{eco} intact (frequency shifts are second order — a fourth predicted null) and acts instead through combination lines. Its first spectral trace is detected in the US output-gap spectrum: a sideband at $\omega_J + \omega_d$ ($p = 0.026$ against a surrogate continuum null, with clean placebos), where ω_d is estimated independently from WPP2024 births. The stationary state of the coupled pair is correlated — two-mode entanglement of the cycle and the pyramid — with the sign observed in the Kuznets-band data.

Several extensions remain. First, the two-oscillator tests should be taken to the full 36-country panel: the sideband detector and the Mathieu-tongue geometry ($2\omega_d$ against each country’s Juglar band) are mechanical to replicate and would turn one marginal US detection into a panel-wide restriction; a full Floquet treatment of the parametric channel is the natural theoretical companion. Second, richer multi-dimensional versions — output, inflation and financial conditions as coupled subsystems — would activate the entanglement machinery beyond two modes. Third, the mean-field non-linearity (Gross–Pitaevskii) was linearized throughout; the self-trapped regime, where a deep recession is sustained by its own pessimism, deserves a treatment of its own. Fourth, the ZLB wall is infinite; a finite wall would accommodate the negative-rate experience of the ECB and SNB via tunnelling. Fifth, the real-time selection of the oscillatory branch could be recast in the intrinsic quantum mean field game formalism of Kolokoltsov (2022).

The deeper point survives all refinements. The mathematics of quantum mechanics is not a metaphor imported into economics; it is the closed form that heterogeneous-agent macroeconomics takes

when its two equations — one running backward from plans, one forward from realizations — are read as a single complex object. We do not claim that the economy is literally quantum-mechanical. *We claim that the mathematics of quantum mechanics is the natural language of aggregate dynamics under heterogeneity, and a better one than the mathematics of Newtonian mechanics.* Just as physicists chose Schrödinger over Newton when the phenomena demanded it, macroeconomists may find value in the same transition.

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